Localizations in coupled electronic chains

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We studied effects of random potentials and roles of electron-electron interactions in the gapless phase of coupled Hubbard chains, using a renormalization group technique. For non-interacting electrons, we obtained the localization length proportional to the number of chains, as already shown in the other approaches. For interacting electrons, the localization length is longer for stronger interactions, that is, the interactions counteract the random potentials. Accordingly, the localization length is not a simple linear function of the number of chains. This interaction effect is strongest when there is only a single chain. We also calculate the effects of interactions and random potentials on charge stiffness.

I. INTRODUCTION

One of the attractive topics in mesoscopic one dimensional (1D) electron systems is the disorder effect. While no extended state can survive in the presence of random potentials in 1D infinite systems, the systems of finite size can be metallic if the size is smaller than the localization length. It is always important to take a serious look at interplay between random potentials and electron-electron interactions when we discuss the transport properties of mesoscopic metallic wires, which are now available as an achievement of the recent technological progress. An interesting example is a mesoscopic metallic ring, which have attracted much attention since a large persistent current was observed even with a modest amount of impurities in the ring [1]. A simple study, only taking an account of the impurity effect, failed to explain such large current [2], and it is necessary to consider electron-electron interactions.

It is in general quite difficult to take correctly account of interactions. One of the exceptions is 1D systems, where interactions can be treated rather rigorously with help of the bosonization techniques. A renormalization group (RG) calculation for 1D systems with impurities was performed by Giamarchi and Schulz [3]. One of their interesting results is that partilce-particle interactions of spinless Fermions would enhance the disorder effect and help the system localize, while the interactions of spinning Fermions (*e.g.* Hubbard type interactions) would counteract the disorder [4].

The RG calculation showed that the effective backward impurity scattering W of a spinless Fermion system is renormalized as $dW/dl = (3 - 2K_\rho)W$ where K_ρ is the Luttinger parameter of the charge mode [3]. The larger repulsive interactions give rise to the smaller K_ρ and therefore to the stronger effective impurity potentials. This is due to the enhancement of charge density wave (CDW) correlations in the ground state of the system. The repulsive interactions would enhance the CDW correlations, which make the system easily pinned by the impurities. For electrons with spins, however, the renormalized impurity potential is given by $\mathrm{d}W/\mathrm{d}l = (3 - K_\rho - K_\sigma - y)W$, where K_ν $(\nu = \rho, \sigma)$ is the Luttinger parameters of charge and spin modes respectively and y measures the backward scattering strength between electrons of opposite spins [3,4]. For Hubbard model with small U, this RG equation becomes

$$\frac{dW}{dl} = \left(1 - \frac{U}{\pi v_F}\right) W, \tag{1}$$

where v_F is the Fermi velocity. Here the electron-electron interactions would screen the impurity potentials. Since a spin density wave (SDW) correlation is dominant in the ground state of the repulsive Hubbard model, the interaction U makes the particle density uniform, and therefore make the coupling of the density to the impurities weak.

These opposite roles of interactions in the spinless and spining Fermions was later checked numerically in connection with the problem of persistent current [5–7]. Although the suppression of the effective disorder strength due to electron-electron interactions in the models with the spin degrees of freedom might explain the observed large persistent current in the disordered metal rings, we should carefully consider the fact that the metal rings have the finite cross section and therefore have the finite number of channels.

Ladder systems have recently attracted a wide range of attentions. Disorder effect on electronic ladder systems is however not widely investigated so far, and there are few studies which calculated RG equations for two-leg electronic ladder with impurity potentials [8–10]. If we just focus on the role of interactions in the presence of impurities, it is shown that the role changes drastically depending on the phase; the impurity effect is enhanced by electron-electron interactions in C1S2 phase (CnSm means that n charge modes and m spin modes are gapless) [8] while it is suppressed in C2S2 phase (with all modes being gapless) [10]. Remember that the ground state of a single chain is gapless and the impurity effect is suppressed by the interactions. So, as far as we concern the gapless phase, the role of interactions

acting against random potentials does not alter even when we increase the number of channels from one to two. But the point we should note is that the suppression of the effective disorder strength due to interactions is smaller in two channel ladder than single channel chain [10]. If this tendency continues in the systems with more channels, we could not expect the large interaction-driven suppression of electron localizations in a thin but finite cross-section wire. It is therefore of importance to see in the increased number of channels whether electron-electron interactions counteract random potentials and if so how large the effect is.

In this paper we first show in Sec. 2 how the disorder effect on a non-interacting electronic ladder changes as the number of chains is increased. In Sec. 3 we turn on electron-electron interactions and investigate the interplay between the interactions and the impurity potentials in one-, two-, and three-chain ladders. Section 4 is devoted for summary of the paper.

II. NONINTERACTING DISORDERED CHAINS

We start from a tight-binding Hamiltonian of coupled chains with open boundary condition,

$$H_0 = -t\sum c^\dagger_{m\sigma i}c_{m\sigma i+1} - t_\perp \sum c^\dagger_{m\sigma i}c_{m+1\sigma i} + h.c., \tag{2}$$

where m and i are the chain and site indices, respectively. This hopping terms are diagonalized by a unitary transformation $c_m = \sum_\alpha V_{m\alpha}a_\alpha$ and they form bands. The matrix V for one, two, three-chain systems, for example, is given by

$$V = \begin{cases} 1 & (N_{ch}=1), \\ \begin{pmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \end{pmatrix} & (N_{ch}=2), \\ \begin{pmatrix} \frac{1}{2} & \frac{1}{\sqrt{2}} & \frac{1}{2} \\ \frac{1}{\sqrt{2}} & 0 & -\frac{1}{\sqrt{2}} \\ \frac{1}{2} & -\frac{1}{\sqrt{2}} & \frac{1}{2} \end{pmatrix} & (N_{ch}=3), \end{cases} \tag{3}$$

where N_{ch} is the number of the chains. Following the standard bosonization procedure, we get the Tomonaga-Luttinger type Hamiltonian

$$H_0 = \sum_{\nu\alpha}\int \frac{dx}{2\pi}[u_{\alpha\nu}K_{\alpha\nu}(\pi\Pi_{\alpha\nu})^2 + \frac{u_{\alpha\nu}}{K_{\alpha\nu}}(\partial_x\phi_{\alpha\nu})^2], \tag{4}$$

where $\nu = \rho, \sigma$ represents charge and spin mode respectively, and α is the band index. With no interaction, the parameters K are equal to 1 and u are the Fermi velocities of each band.

The impurity scattering potential term is originally written in the form

$$H_{imp} = \sum n^{imp}_{mi}c^\dagger_{m\sigma i}c_{m\sigma i} \tag{5}$$

$$= \sum n^{imp}_{mi}V^*_{m\alpha}V_{m\beta}a^\dagger_{\alpha\sigma i}a_{\beta\sigma i}, \tag{6}$$

where V^* is the complex conjugate of V. After the bosonization we get the forward and backward scattering terms. Since the forward scattering does not affect H_0 [9], we only consider the effect of the backward impurity scattering. The backward scattering terms are given by

$$H_b = \frac{1}{\pi a}\int dx \sum_{\alpha\beta,s}\xi_{\alpha\beta}e^{-i\{\phi_{\alpha\rho}+\phi_{\beta\rho}+s(\phi_{\alpha\sigma}+\phi_{\beta\sigma})\}/\sqrt{2}}\cos[\{(\theta_{\alpha\rho}-\theta_{\beta\rho})+s(\theta_{\alpha\sigma}-\theta_{\beta\sigma})\}/\sqrt{2}] + h.c, \tag{7}$$

where a is the lattice constant. ξ is assumed to be a linear combination of Gaussian random variables and then the following general form of the action will appear in the replica trick method by integrating out the random variables,

$$S^{imp}_b = \frac{2}{(\pi a)^2}\sum_{\alpha\beta\gamma\delta}\sum_{ijss'}Z^{\alpha\beta}_{\gamma\delta}\int \cos\{(\theta^i_{\alpha\rho}-\theta^i_{\beta\rho}+s(\theta^i_{\alpha\sigma}-\theta^i_{\beta\sigma}))/\sqrt{2}\}\cos\{(\theta^j_{\gamma\rho}-\theta^j_{\delta\rho}+s'(\theta^j_{\gamma\sigma}-\theta^j_{\delta\sigma}))/\sqrt{2}\} \tag{8}$$

$$\times\cos[\{(\phi^i_{\alpha\rho}+\phi^i_{\beta\rho}+s(\phi^i_{\alpha\sigma}+\phi^i_{\beta\sigma}))-(\phi^j_{\gamma\rho}+\phi^j_{\delta\rho}+s'(\phi^j_{\gamma\sigma}+\phi^j_{\delta\sigma}))\}/\sqrt{2}],$$

where i, j are the replica indices. In order to discuss the disorder effect of various coupled chains on the same basis, we assume n_{mi}^{imp} is a Gaussian random variable and satisfies $< n_{mi}^{imp} n_{lj}^{imp} >= W\delta_{ml}\delta_{ij}$. Hence we have

$$Z_{\gamma\delta}^{\alpha\beta} = W \sum_n V_{n\alpha}^* V_{n\beta} V_{n\gamma} V_{n\delta}^*. \tag{9}$$

The RG equation for $Z_{\gamma\delta}^{\alpha\beta}$ is

$$\frac{dZ_{\gamma\delta}^{\alpha\beta}}{dl} = Z_{\gamma\delta}^{\alpha\beta}, \tag{10}$$

where we used the fact that K's are all equal to 1 for free particles. The solution of the RG equation is $Z_{\gamma\delta}^{\alpha\beta}(l) = e^l W \sum_n V_{n\alpha}^* V_{n\beta} V_{n\gamma} V_{n\delta}^*$. Note that $We^l = \sum_\beta Z_{\alpha\beta}^{\alpha\beta}(l)$. The RG equation stops when $Z_{\gamma\delta}^{\alpha\beta}(l)$ reaches $\sim \overline{v_F}^2/a$, and it is when $l \sim \log(L_{loc}/a)$ where L_{loc} is the localization length. Putting all together, we get

$$L_{loc} \sim N_{ch} \frac{\overline{v_F}^2}{W}. \tag{11}$$

On the other hand, the electron scattering rate is given by $\tau^{-1} \sim W\rho(\epsilon_F)$, where $\rho(\epsilon_F)$ $(\sim \overline{v_F^{-1}})$ is the density of states at the Fermi level, and hence the mean free path $l_e = \overline{v_F}\tau \sim \overline{v_F}^2/W$. Then we get the known relation [11,12], $L_{loc} \sim N_{ch} l_e$.

In order to see the disorder effect on the transport properties, we next calculate charge stiffness D, which measures the strength of Drude peak, $\sigma = D\delta(\omega) + \sigma_{reg}$. Note that D is also a measure of the persistent currents for small flux.

The external flux couples with the current $\sum_{\alpha s} j_{\alpha s}$, where α is the band index. Since $\sum_{\alpha s} j_{\alpha s} \propto \sum_\alpha \Pi_{\alpha\rho}$, charge stiffness is proportional to $\sum_\alpha K_{\alpha\rho} u_{\alpha\rho}$. Ignoring the irrelevant numerical factor, we define charge stiffness per channel by $D = (1/N_{ch}) \sum_\alpha K_{\alpha\rho} u_{\alpha\rho}$. For non-interacting and non-impurity ladder, D is the averaged Fermi velocity $\overline{v_F}$.

In the presence of impurities the RG equation for the charge stiffness is given by

$$\begin{aligned} \frac{dD}{dl} &= -\sum_\alpha \frac{2a u_{\alpha\rho}^2}{u_{\alpha\sigma}^3 \pi} \left(\frac{1}{N_{ch}} \sum_\beta Z_{\alpha\beta}^{\alpha\beta}\right) \\ &= -\frac{2a}{\pi} \overline{v_F^{-1}} W e^l. \end{aligned} \tag{12}$$

D has no explicit N_{ch} dependence and the effect of increasing channel number could appear only through $\overline{v_F^{-1}}$, which is usually weak. For a finite system of size L, D is given by

$$D \sim D_0 - \frac{2}{\pi} \overline{v_F^{-1}} W L, \tag{13}$$

where D_0 is a constant. For a given size L, the charge stiffness D is smaller for stronger disorder, and for a given disorder W, D is smaller for a larger system. The former is trivial, and the latter is because L_{loc}/L becomes smaller as L increases with W fixed and hence the system will be in more localized regime. This will be clear if we rewrite Eq.(13) as

$$D \sim D_0 - \frac{2}{\pi} N_{ch} \overline{v_F} \frac{L}{L_{loc}}. \tag{14}$$

We cannot simply extend these discussion to $N_{ch} \to \infty$, that is, to two dimensions (2D), because we constructed the RG theory based on the Tomonaga-Luttinger type bosonized Hamiltonian with well-defined subbands, which is not an appropriate basis for 2D system.

III. INTERACTING DISORDERED CHAINS

In the previous section we showed that the localization length is proportional to the number of chains, and the effect of random potentials becomes weaker as the chain number increases. What happens if we turn on interactions between the particles. The original Hamiltonian is then Eq.(2) plus the Hubbard-type interaction term,

$$U\sum n_{m\uparrow i}n_{m\downarrow i}. \tag{15}$$

A part of the interaction terms can be combined with the kinetic term to give

$$H_0=\sum_{r\nu}\int\frac{dx}{2\pi}[u_{r\nu}K_{r\nu}(\pi\Pi_{r\nu})^2+\frac{u_{r\nu}}{K_{r\nu}}(\partial_x\phi_{r\nu})^2], \tag{16}$$

where $\nu=\rho,\sigma$ represents charge and spin mode respectively, and r is the new band index assigned by the unitary transformation $\phi_{\alpha\nu}\to\sum_r T_{\alpha r}\phi_{r\nu}$ and $\Pi_{\alpha\nu}\to\sum_r T_{\alpha r}\Pi_{r\nu}$, where

$$T=\begin{cases}1 & (N_{ch}=1),\\ \begin{pmatrix}\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}}\\ \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}}\end{pmatrix} & (N_{ch}=2),\\ \begin{pmatrix}\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{6}}\\ 0 & \frac{1}{\sqrt{3}} & -\sqrt{\frac{2}{3}}\\ -\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{6}}\end{pmatrix} & (N_{ch}=3).\end{cases} \tag{17}$$

This unitary transformation is necessary to diagonalize H_0. Since a part of interaction terms are included in H_0, the parameters K and u have different values from the ones of the noninteracting case. Hereafter we use Greek letters $\alpha,\beta,\cdots$ for the old band index and Roman letters $r,\cdots$ for the new band index.

The other remaining interaction terms are represented by H_1. Since there are still a lot of terms in H_1, we do not write them down all here and we only note that the interaction matrix element $g_{\alpha\beta\gamma\delta}$ in the commonly used notation, representing the scattering from (α,β) bands to (γ,δ) bands, is given by

$$g_{\alpha\beta\gamma\delta}=U\sum_l V_{l\alpha}V_{l\beta}V^*_{l\gamma}V^*_{l\delta} \tag{18}$$

in the case of Hubbard model, although some of them, such as $g_{\alpha\beta\beta\alpha}$ and $g_{\alpha\beta\beta\beta}$, should be set to zero due to lack of the momentum conservation. As stressed in Ref. [13], one should not forget the sign factors of g which come from Majorana Fermion operators. In the present case, the backward scattering interaction $g^{(1)}_{\alpha\alpha\beta\beta}$, for example, has to be multiplied by -1.

The presence of the interaction terms made the phase diagram quite rich [13–21], and the interplay between interactions and random potentials is strongly phase dependent as stated in Sec. 1. In order to construct a systematic view with changing the number of chains, we should pick up a common phase, *gapless* phase, where no gap opens both in the charge and spin modes. There is also another reason to study gapless phase. When the forward impurity scatterings are considered, Fujimoto and Kawakami showed [9] that the electron-electron interactions are effectively suppressed and as a result the charge and spin gaps collapse. In this situation the remaining task is to study the effect of the backward impurity scatterings on the gap-collapsed (gapless) phase.

In the gapless phase the action presenting the backward impurity scatterings is again given by Eq.(8). The RG equation for $Z^{\alpha\beta}_{\gamma\delta}$ is

$$\begin{aligned}\frac{dZ^{\alpha\beta}_{\gamma\delta}}{dl}=&[3-\frac{1}{8}\sum_r\{\{(T_{\alpha r}-T_{\beta r})^2+(T_{\gamma r}-T_{\delta r})^2\}(K^{-1}_{r\rho}+K^{-1}_{r\sigma})\\ &+\{(T_{\alpha r}+T_{\beta r})^2+(T_{\gamma r}+T_{\delta r})^2\}(K_{r\rho}+K_{r\sigma})\}]Z^{\alpha\beta}_{\gamma\delta}\\ &-\frac{1}{u_\sigma\pi}\sum_{\alpha'\beta'}g^{(1)}_{\alpha'\gamma\delta\beta'}Z^{\alpha\beta}_{\alpha'\beta'},\end{aligned} \tag{19}$$

where $g^{(1)}$ represents the backward scattering interactions. Note that, for noninteracting electrons, we have $K=1$ and $g=0$ and then Eq. (19) reduces to Eq. (10). Also, when $N_{ch}=1$, Eq. (19) reduces to $dW/dl=(1-U/u_\sigma\pi)W$ which is just what we mentioned in Eq. (1).

Since the renormalization of $Z^{\alpha\beta}_{\gamma\delta}$ has a strong K dependence, the localization length and other physical quantities would changes in accordance with K in a complicated way. To make the story simple, we consider the weak interaction

limit and use $K = 1$ as in the noninteracting system. Also we assume the random impurity potentials are so weak that we can forget about the renormalization of K. The RG equation (19) can then be written in the following matrix form,

$$\frac{dZ}{dl} = (1 - G)Z. \tag{20}$$

The (i, j) element of the $N_{ch} \times N_{ch}$ matrix Z is $Z^{\alpha\beta}_{\gamma\delta}$ where $i = (\alpha, \beta)$ and $j = (\gamma, \delta)$. The (i, j) element of the matrix G is given by $\frac{1}{u_\sigma \pi} g^{(1)}_{\gamma\alpha\beta\delta}$ where $g^{(1)}$ is now assumed to include the sign created by the Majorana Fermion operators mentioned above. The signs of the elements of G are not always plus and it is not in general trivial whether the presence of G in Eq. (20) would weaken the growth of Z, namely, whether the interactions would suppress the impurity effects. So let us see it in more detail.

Since we assume the interactions are weak and ignore the renormalization of G, the solution of Eq. (20) is $Z(l) = e^{(1-G)l} Z(0) \sim e^l (1 - Gl) Z(0)$. Therefore $Tr(Z(l)) \sim e^l \{N_{ch} W - l Tr(GZ(0))\} \sim N_{ch} W e^{(1-CU/\overline{v_F}\pi)l}$, where $C = u_\sigma \pi Tr(GZ(0))/(N_{ch} WU)$. Since, when $l \sim \log(L_{loc}/a)$, $Tr(Z(l)) \sim N_{ch}^2 \overline{v_F}^2/a$, we have $N_{ch}\overline{v_F}^2/Wa \sim (L_{loc}/a)^{1-CU/\overline{v_F}\pi}$. Then the localization length of the interacting system has the form,

$$\frac{L_{loc}}{a} \sim \left(\frac{L^{(0)}_{loc}}{a} \right)^{1+CU/\overline{v_F}\pi}, \tag{21}$$

where $L^{(0)}_{loc} = N_{ch}\overline{v_F}^2/W$ is the localization length of the noninteracting system. Because of the presence of U, the localization length is no longer simply proportional to N_{ch}. If we forget about the sign of $g_{\alpha\beta\gamma\delta}$ created by the Majorana Fermion algebra, discussed above, $C = 1$ for any N_{ch}. Taking the sign into account, however, C becomes

$$C = \begin{cases} 1 & (N_{ch} = 1), \\ 0.25 & (N_{ch} = 2), \\ 0.4 \cdots & (N_{ch} = 3). \end{cases} \tag{22}$$

Therefore the interaction U always makes the localization length longer, namely, the interactions have a delocalization effect in the gapless phase of the coupled chain systems. C is not a monotonic function of N_{ch} as far as $1 \le N_{ch} \le 3$. and hence it is not always true that the interaction effect becomes weaker as N_{ch} increases. Anyway, since it is easy to prove $C \le 1$, the delocalization effect is strongest when $N_{ch} = 1$.

Next we study the charge stiffness D. As noted in the previous section, the external flux couples to $\sum_{\alpha s} j_{\alpha s} \propto \sum_\alpha \Pi_{\alpha\rho} = \sum_{\alpha r} T_{\alpha r} \Pi_{r\rho}$. Recall that the Greek letter α represents the old band index assigned before the operation of T, and the Roman letter r is the new band index. Since $\sum_\alpha T_{\alpha r} = 1$ (when $N_{ch} = 1$), $\sqrt{2}\delta_{r1}$ (when $N_{ch} = 2$), $\sqrt{3}\delta_{r2}$ (when $N_{ch} = 3$), the charge stiffness is given by $D = u_{r\rho} K_{r\rho}$ where $r = 1$ (when $N_{ch} = 1, 2$), $r = 2$ (when $N_{ch} = 3$) with the irrelevant numerical factor being omitted. The RG equation is then given by

$$\begin{aligned} \frac{dD}{dl} &= -\frac{2a u_{r\rho}^2}{u_{r\sigma}^3 \pi} \sum_{\alpha\beta} Z^{\alpha\beta}_{\alpha\beta} T^2_{\alpha r} \\ &\sim -\frac{2a}{\pi} \overline{v_F}^{-1} W e^{(1-CU/\overline{v_F}\pi)l}, \end{aligned} \tag{23}$$

where $r = 1$ (when $N_{ch} = 1, 2$), $r = 2$ (when $N_{ch} = 3$). In the second line of Eq. (23) the interactions are assumed to be small. For a finite system of size L,

$$D \sim D_0 - \frac{2a}{\pi} \overline{v_F}^{-1} W \left(\frac{L}{a} \right)^{1-CU/\overline{v_F}\pi}. \tag{24}$$

Again, the interaction U suppresses the random potential effect and gives larger value of the charge stiffness that in the noninteracting case.

IV. SUMMARY

We investigated the role of electron-electron interactions in the coupled Hubbard chains with random potentials. For noninteracting systems, a RG calculation shows that the effective strength of the impurity potentials grows towards

the strong coupling limit and the localization length is proportional to the number of chains N_{ch}. In the presence of interactions between the particles, the role of the interactions changes from the phase to phase as previously shown in Ref. [8,10], and therefore we only focused on the gapless phase of N_{ch} chain systems. Based on the RG calculation, we showed that the interactions always reduce the effective strength of the impurity potentials. The degree of the reduction is sensitive to N_{ch}, and the counter-effect of the interactions against the random potentials is strongest when $N_{ch} = 1$.

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